

Adaptive Learning and Survey Data

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March 27, 2014

Abstract

This paper investigates the ability of the adaptive learning approach to replicate the expectations of professional forecasters. For a range of macroeconomic and financial variables, we compare constant and decreasing gain learning models to simple, yet powerful benchmark models. We find that constant gain models provide a better fit for the expectations of professional forecasters. For macroeconomic series they usually perform significantly better than a naïve random walk forecast. In contrast, we find it difficult to beat the no-change benchmark using the adaptive learning models to forecast financial variables.

Keywords: expectations, survey of professional forecasters, adaptive learning, bounded rationality

JEL Codes: E37, E44, G14, G15

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1 Introduction

Expectations are a key ingredient of many economic and financial models. They reflect behavior of agents and influence economic outcomes. The macroeconomic and finance literature usually endows agents with rational expectations. Although unlikely in reality, they present some advantages over bounded rationality. Under rational expectations, agents' subjective probability distribution coincides with the true distribution of the model economy. This model consistency makes rational expectations unique to the model. In contrast, there is an infinite number of non-rational ways to form expectations.

One widely cited alternative to rational expectations has been suggested by Bray (1982), Bray and Savin (1986), Marcet and Sargent (1989) and Sargent (1993). This approach assumes that agents, modeled by economists, have at most the same knowledge as these economists themselves and hence they behave as econometricians. This approach, called adaptive learning, has the advantage of imposing modeling discipline. Similar to rational expectations, adaptive learning generates forecasts which are optimal given agents' information at the time.^{1,2}

In this paper, we systematically test for the empirical validity of this approach. Specifically, we investigate in how far agents' expectations can be approximated by adaptive learning. For this purpose, we use a set of financial and macroeconomic survey data and fit adaptive learning laws of motions. The formal test of this approach is conducted by assessing the out-of-sample forecasting performance of such estimated models.

We assume that economic agents use simple time series models to make forecasts and they update their parameters using recursive least squares.³

¹By optimal we mean here that the forecasts are orthogonal to forecast errors.

²There is an extensive literature that uses adaptive learning to understand numerous unexplained, stylized facts in finance and macroeconomics. For example, Adam et al. (2009) and Evans and Branch (2010) who show that adaptive learning helps in replicating a variety of asset pricing puzzles. Milani (2011) and Preston and Eusepi (2011) argue that learning dynamics can explain business cycle fluctuations.

³Numerous empirical studies have demonstrated that fundamentals-based models can frequently underperform forecasts derived from simple univariate time series models. For instance, Orphanides and Van Norden (2005) show that autoregressive models generate better forecast than the Phillips curve-based models while Stock and Watson (2007)

We consider simple time-series models, where the parameters are updated by recursive least squares with constant and decreasing gains, adaptive expectations and the random walk model.⁴ Additionally, for each variable we use macroeconomic and financial regressors.

We first estimate parameters in an initial training sample and assess their forecasting accuracy out-of-sample. We also conduct forecast accuracy tests against the simple, yet powerful benchmark of the random walk. Additionally, we evaluate constant gain specifications against decreasing gain models and against constant parameter models.

We find that most models have very low estimated gain parameters, which suggests that agents use a relatively long history of observations to make their forecasts. Out-of-sample Diebold-Mariano tests show that for all macroeconomic series at least one model performs significantly better than a naïve, random walk forecast. This suggests that, instead of relying only on the last available observation, agents use more complex models to form their expectations about the behavior of macroeconomic variables. In contrast, among the financial variables' forecasts only the first order autoregressive specification for Yen-U.S. Dollar exchange rate significantly beats the no-change benchmark. The comparison between constant gain and decreasing gain specifications suggests that the former approximates survey expectations better.

A large body of literature in macroeconomics and finance employed survey data to examine expectation formation by economic agents. This literature can be broadly divided in two strands: studies testing rational expectations hypothesis and studies proposing alternative modeling approaches to match survey expectations. The first branch consists of a group of studies seeking to verify the hypothesis of rationality for inflation survey expectations, including Bonham and Dacy (1991), Bonham and Cohen (2001), Croushore (1997), and Evans and Gulamani (1984). This body of research largely documents the failure of the rational expectations hypothesis for

demonstrate that an IMA model provides better forecasts for inflation. Meese and Rogoff (1983) and Cheung, Chinn, and Pascual (2005) demonstrate forecasting superiority of random walk over fundamentals-based models for exchange rates.

⁴A recent trend in the literature on forecasting is the use of model combinations as surveyed by Timmermann (2006). This is, however, beyond the scope of the current work.

inflation expectations.⁵

Similar results have been found using survey expectations of foreign exchange market traders by Frankel and Froot (1987b, 1990a) and Ito (1990). Froot (1989), Friedman (1990), and Jeong and Maddala (1996), who also reject the hypothesis of rationality for interest rate forecasts. More recently, Bacchetta, Mertens and van Winkoop (2009) investigate the link between the predictability of excess returns and expectational errors in a set of financial markets using survey data. They find predictability of excess returns and expectational errors in foreign exchange, stock and bond markets.

As survey data reject the rational expectations hypothesis for a large number of markets, new ways of modeling agents' beliefs have been proposed. Allen and Taylor (1990), Ito (1990), and Frankel and Froot (1987a, 1990a, and 1990b) argue that the dynamics of survey expectations of foreign exchange traders display behavioral rather than rational features. Specifically, they found forecast heterogeneity across individuals and over time. Ito (1990) argues that, in line with behavioral bias of 'wishful thinking', exporters tend to anticipate a currency depreciation while importers anticipate an appreciation. Frankel and Froot (1987a, 1990a, and 1990b) and Taylor and Allen (1992) show that, at short horizons, traders tend to use extrapolative chartist rules, whilst at longer horizons they tend to use mean reverting rules based on fundamentals.

The time-varying heterogeneity of expectations have been modeled by Brock and Hommes (1997) within the framework of an adaptively rational equilibrium dynamics (ARED). Branch (2004) uses ARED to test whether inflation survey data exhibited rationally heterogeneous expectations. Similarly, Jongen, Verschoor, Wolff, and Zwinkels (2012) explained dispersion in survey data on currency movements using the ARED framework.

The average dynamics displayed by economic agents' beliefs have often been modeled by the adaptive learning mechanism, which presumes that agents behave as econometricians in the sense that they estimate model parameters using econometric techniques (for example, Evans and Honkapohja,

⁵An important exception is the work by Keane and Runkle (1990), which fails to reject the rational expectations hypothesis.

2001). Numerous applications and extensions of standard adaptive learning have been suggested in the recent literature. Initially, recursive least squares parameter learning, as proposed by Bray (1982) and Marcet and Sargent (1989), suggested small departures from rationality by assuming that economic agents knew the model of the economy and updated only its parameters. Additional extensions of this approach include constant gain learning that occasionally leads to escape dynamics (Cho, Williams and Sargent, 2002).

This paper is closely related to the previous studies testing adaptive learning using survey data. Branch and Evans (2006) estimate various laws of motion for expectation formation and find that the constant gain learning rule fits best in sample and performs well in an out-of-sample exercise. Pfajfar and Santoro (2010) identify the degree of heterogeneity and the forecasting rules of private agents' forecasts of inflation using survey expectations. Berardi and Galimberti (2012b) assess the empirical plausibility of least squares and stochastic gradient algorithms using survey expectations on US inflation and output growth. All of these studies find some support for adaptive learning behavior in survey data.

While the previous studies provide empirical tests of adaptive learning for macroeconomic series, our aim is to cover a larger set of variables and markets, in particular financial markets, to provide a systematic overview of agents' beliefs. Furthermore, we use real time data, which represent the information set available to the professional forecasters.

The remainder of the paper is organized as follows. We describe the models of expectations and their estimation in Section 2. Section 3 discusses the data. The results of the forecasting exercise are presented in Section 4. Robustness checks to some important assumptions is provided in Section 5. Finally, Section 6 concludes.

2 Modeling expectations

Rational forecasts are built on strong assumptions about the representative agent's information set. We relax these informational assumptions by pos-

tulating that, instead of using economic model based forecasts, agents use simple time-series models to make predictions. This type of forecasting rule embodies very low computational cost as it is based on the past values of observed variables only. Agents are assumed to update parameters of their forecasting rules using recursive least squares.

2.1 Recursive least squares

The forecasting rule is governed by adaptive learning where agents behave as econometricians and, for a linear model, $y_t = \boldsymbol{\theta}_t \mathbf{x}_t + \varepsilon_t$, they update model parameters using recursive least squares,

$$\begin{aligned}\boldsymbol{\theta}_t &= \boldsymbol{\theta}_{t-1} + \gamma_t \mathbf{R}_t^{-1} \mathbf{x}_t (y_t - \boldsymbol{\theta}'_{t-1} \mathbf{x}_t), \quad t = 1, 2, \dots, T, T+1 \\ \mathbf{R}_t &= \mathbf{R}_{t-1} + \gamma_t (\mathbf{x}_t \mathbf{x}'_t - \mathbf{R}_{t-1})\end{aligned}\quad (1)$$

where $\boldsymbol{\theta}_t$ is a $k \times 1$ vector of parameters, \mathbf{x}_t is the $k \times 1$ vector of regressors, γ_t is the gain parameter, and \mathbf{R}_t is the second moment matrix.

The gain parameter γ_t can take two forms. First, when $\gamma_t = 1/t$, the estimator converges to the OLS estimator. The finite-sample difference between (1) and the OLS estimator is determined by the initialization of \mathbf{R}_0 , which we set to $\mathbf{0}$ to obtain OLS forecasts from expanding windows. Under parameter stability OLS forecasts are optimal in the mean square forecast error (MSFE) sense. Since the weight associated with an observation decreases with the amount of available data, this learning scheme is referred to as “decreasing gain” learning and the estimator is referred to as “decreasing gain least squares” (DGLS).

Second, when the parameter γ_t is set to a constant, $\gamma_t = \gamma$, $\forall t$ with $\gamma \in (0, 1]$, observations receive decaying weights that decrease with the distance from the most recent observation. This updating rule is known as a “constant gain learning” or “perpetual learning” and the estimator is referred to as “constant gain least squares” (CGLS). CGLS can be shown to deliver good forecasts under parameter instability. In a simplified model (adaptive expectations, discussed below), it can be shown to be the optimal forecasting strategy in the MSFE sense when the model’s parameter evolves

as a random walk (Pesaran, Pick and Pranovich, 2013).⁶

The vector of parameters, $\boldsymbol{\theta}_t^{(i)}$, is used to make forecast for the next period

$$\hat{y}_{t+1}^{(i)} = \boldsymbol{\theta}_t^{(i)'} \mathbf{x}_{t+1}$$

where the superscript $i \in \{\text{dg}, \text{cg}\}$ denotes the particular forecasting scheme, and $\hat{y}_{t+1}^{(\text{dg})}$ is the forecast using a decreasing gain and $\hat{y}_{t+1}^{(\text{cg})}$ that using a constant gain.

We use a range of regressor sets. First, we experiment with autoregressive specifications, in particular, AR(1) to AR(4). Additionally, we include one or more economic variables that may improve forecasts with details provided below. The final regressor set we consider contains only the intercept. When using only an intercept, that is, $x_t = 1, \forall t$, the expectation of the dependent variable is the intercept, $\hat{y}_{t+1} = \theta_t$.

Setting $R_0 = 1$, it is easy to see that CGLS reduces to what is called “adaptive expectations” or “exponential smoothing”:

$$\hat{y}_{t+1}^{(a)} = \gamma y_t + (1 - \gamma) \hat{y}_t^{(a)} \quad (2)$$

where superscript (a) denotes the forecast from adaptive expectations. As already mentioned, this specification has optimality properties under certain data generating processes. Furthermore, in practice, it has been shown to have good forecasting properties for a wide range of variables (Hyndman, Koehler, Ord and Snyder, 2008, Pesaran and Pick, 2011, Pesaran, Pick and Pranovich, 2013).

Under decreasing gain learning and setting $y_0 = 0$, adaptive expectations yield the simple mean forecast

$$\hat{y}_t = t^{-1} \sum_{s=1}^t y_s$$

Finally, when setting the downweighting parameter, γ , to unity, adaptive expectations (2) yields the random walk forecast

$$\hat{y}_{t+1}^{(\text{RW})} = y_t \quad (3)$$

⁶The optimality of OLS and adaptive expectations, naturally, requires that the specified model is the correct one.

which the theoretical literature on expectations calls “naïve expectations”.

When a random walk time series is differenced, the optimal forecast is

$$\hat{y}_{t+1}^{(\Delta\text{RW})} = 0 \quad (4)$$

The forecasts generated by adaptive learning will be compared to both random walk models as it is well known that many macroeconomic time series are close to the random walk specification (3) whereas many financial returns are close to specification (4).

2.2 Estimation of the model parameters

The specifications of the constant gain least square forecasting rule and the adaptive expectations forecasting rule require estimates of the gain parameter, γ , and initialization of the time varying parameters. We seek to identify the law of motion (within a certain class of models) that professional forecasters use. We assume that they employ only observed data to make their forecasts. Accordingly, the parameter of the forecasting rule, γ , is estimated by minimizing the mean square error between the forecasts resulting from the gain parameters and the survey forecasts:

$$\hat{\gamma}^{(i)} = \arg \min_{\gamma \in (0,1]} \left[\frac{1}{T_{\text{init}}} \sum_{t=1}^{T_{\text{init}}} \left(\hat{y}_t^{(i)} - y_t^{(\text{spf})} \right)^2 \right] \quad (5)$$

where superscript i denotes the different forecast specifications, and $y_t^{(\text{spf})}$ is the forecast provided by professional forecasters. We use the forecasts for observations up to T_{init} to estimate the constant gain value, where we set $T_{\text{init}} = 60$.

We use diffuse priors for the parameters at time $t = 0$, that is, we set $\theta_0^{(i)} = \mathbf{0}$ and $\left(\mathbf{R}_0^{(i)} \right)^{-1} = \kappa \left(\mathbf{X}_{1:20}^{(i)'} \mathbf{X}_{1:20}^{(i)} \right)^{-1}$, where $\mathbf{X}_{1:20}^{(i)}$ is the regressor matrix for model i over the first 20 observations and $\kappa = 1000$. This choice of the second moment matrix implies that the variance of the initial parameter set is large, independent of the scale of the data. For the decreasing gain forecasts we estimate parameters by expanding window OLS. We will investigate the robustness of our results to these choices in Section 5.

We use the observations after T_{init} until the end of the sample to construct out-of-sample forecasts, which we will use to evaluate the different

learning models against the median survey of professional forecasters. In order to evaluate the forecasts, we use the MSFE between the median forecasts of the survey of professional forecasters and the forecasts we obtain from each model

$$\text{MSFE} = \frac{1}{T - T_{\text{init}}} \sum_{t=T_{\text{init}}}^T \left(e_t^{(i)} \right)^2 \quad (6)$$

where

$$e_t^{(i)} = \hat{y}_t^{(i)} - y_t^{(\text{spf})}$$

Additionally, we test for predictive accuracy using the Diebold and Mariano (1995) test statistic for the loss differential

$$l_1(i) = \left(e_t^{(\text{RW})} \right)^2 - \left(e_t^{(i)} \right)^2 \quad \text{and} \quad l_2(i) = \left(e_t^{(\Delta\text{RW})} \right)^2 - \left(e_t^{(i)} \right)^2$$

where $e_t^{(\text{RW})}$ and $e_t^{(\Delta\text{RW})}$ are the forecast errors of the random walk specifications in (3) and (4). Additionally, we compare the CGLS forecasts to those obtained from DGLS and to those when using constant parameters from the initial estimation sample, $\theta_{T_{\text{init}}}^{(i)}$, throughout the forecast period

$$\hat{y}_{t+1}^{(i)} = \theta_{T_{\text{init}}}^{(i)'} \mathbf{x}_{t+1}$$

for $t > T_{\text{init}}$. This yields the following loss differentials

$$l_3(i) = \left(e_t^{(\text{CGLS},i)} \right)^2 - \left(e_t^{(\text{DGLS},i)} \right)^2 \quad \text{and} \quad l_4(i) = \left(e_t^{(\text{CGLS},i)} \right)^2 - \left(e_t^{(\text{CP},i)} \right)^2$$

where $e_t^{(\text{CP},i)}$ is the forecast error of the specification with constant parameters.

2.3 Forecasting models

The forecasts from recursive least squares use a number of different models. The first ones are pure autoregressive models, where we have $\mathbf{x}_t = (1, y_{t-1})$ for AR(1) model and up to $\mathbf{x}_t = (1, y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4})$ for the AR(4) model. We also use a range of economic variables in the forecasts, and when added to autoregressive models we use the standard time series notation ARX. When forecasting the inflation rate we use the unemployment rate as a predictor, following the literature on the Phillips curve (for example,

Table 1: Summary of the data

Variable	Explanatory variables	Freq.	Sample	1st fore	no.fore
Inflation	Unemployment	Q	1981.1–2012.3	1996.4	64
3 month T-bill yield	Unemployment, inflation	Q	1981.1–2012.3	1996.4	64
10 year gov't bond yield	Unemployment, inflation	Q	1992.1–2012.3	2007.2	22
Corporate bond yield	Unemployment, inflation, corporate profits	Q	1981.1–2012.3	1996.4	64
GDP growth	10 year gov't bond rate	Q	1970.2–2012.3	1985.2	110
Unemployment	GDP growth rate	Q	1969.1–2012.3	1984.1	115
EUR/USD	Inflation differential, interest differential	M	1996.4–2009.11	2001.4	104
JPY/USD	Inflation differential, interest differential	M	1996.4–2009.11	2001.4	104
GBP/USD	Inflation differential, interest differential	bi-M	1996.5–2009.11	2006.5	22

Note: The first column gives the variables and the second column reports the variables that we use as structural variables in the forecasting exercise. The frequencies are in column headed 'Freq.' are quarterly (Q), monthly (M), and bi-monthly (bi-M). The column headed 'Sample' reports the sample used for estimation and forecasting after taking account of the presample necessary for differencing and lags. The column with heading '1st fore' shows the first period for which an out-of-sample forecast is made. Finally, the column with heading 'no.fore' gives the number of out-of-sample forecasts per variable.

Atkeson and Ohanian, 2001). We add the ten year government bond yield to the model for the real GDP growth following the literature on the predictive power of asset prices for the business cycle (Stock and Watson, 2003). In the regressions for the interest rates we incorporate the unemployment rate and the inflation rate in the spirit of Taylor rule forecasts. For the corporate bond yield we additionally include aggregate corporate profits growth as a regressor. Real GDP growth is added to the regressions for the unemployment growth rate as an indicator of the business cycle. In the exchange rate specifications we incorporate inflation differentials and one month interbank interest differentials. These variables are derived from exchange rate parities: uncovered interest rate parity and purchasing power parity. All the regressors are reported in Table 1.

3 Data

We use a range of macroeconomic and financial data on monthly and quarterly frequencies. The variables are reported in the first column of Table 1. The quarterly data are the U.S. CPI inflation rate, the three month U.S. T-bill rate, the ten year U.S. government bond rate, the Moody's AAA corporate bond yield, the U.S. real GDP growth rate, and the U.S. unemployment rate. The survey of professional forecasters (SPF) data for these series were obtained from the Philadelphia Fed's Real Time Data Research Center and we use the median response in our analysis.⁷

Compared to the mean forecast, the median forecast has the advantage that it is robust to extreme forecasts. Evidence of the existence of outlier forecasts can be obtained from the skewness of the distribution of responses. Plots provided in Appendix A show that on average the skewness is close to zero, so that on average the mean and the median forecasts are similar. However, all time series contain periods where the skewness is large, which implies that extreme forecasts are in the set of responses. These episodes suggest that the robustness properties of the median are useful when considering the aggregate forecasts.⁸

For actual observations on U.S. real GDP and the U.S. unemployment rate we use real time data also obtained from the Philadelphia Fed's Real Time Data Research Center. The remaining quarterly data are from the St. Louis Fed FRED database.

We transform the interest rates, real GDP and the unemployment rate into annual growth rates

$$y_{jt} = \ln(Y_{jt}/Y_{j,t-4})$$

where lower case variables are in growth rate and an upper case variables are level variables, j indicates the variable, and t time. We also transform survey data into expected growth rates

$$y_{jt}^{(\text{spf})} = \ln(Y_{jt}^{(\text{spf})}/Y_{j,t-4})$$

⁷We tested the SPF series for unit roots using the ADF test. All series appear stationary. The results are presented in the online appendix.

⁸For this reason the median is the common measure used in the literature, see, e.g., Osterberg (2000), Carroll (2003), and Mankiw, Reis and Wolfers (2004).

For applications with the real time data, to calculate growth rates, we use the vintages available to the forecasters at the time of the forecast. After allowing for pre-samples to calculate growth rates and for four lags for the autoregressive models, we are left with the samples' lengths reported in the fourth column of Table 1.

The variables used as economic regressors are reported in the second column of Table 1. An additional variable is the growth rate of U.S. corporate profits, which we use as an explanatory variable for the corporate bond yields. This series is from the St. Louis Fed FRED data base.

In addition, we analyze three exchange rate series: the euro-U.S. dollar exchange rate (EUR/USD), the Japanese yen-U.S. dollar exchange rate (JPY/USD), and the British pound-U.S. dollar exchange rate (GBP/USD). We use survey data collected by Consensus Economics of London, and their data set also provides the actual exchange rates. We transform exchange rate series into monthly growth rates

$$y_{jt} = \ln(Y_{jt}/Y_{j,t-1})$$

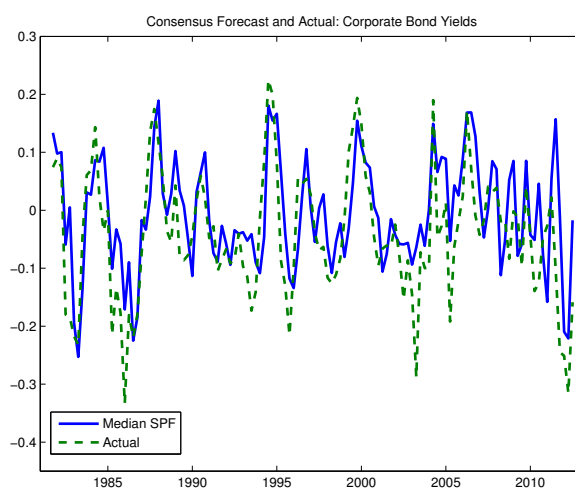
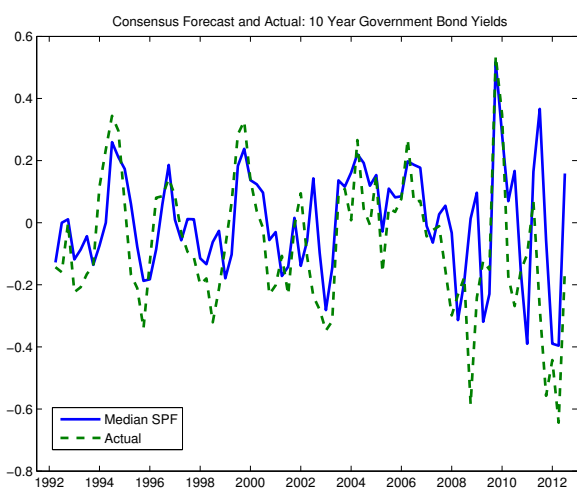
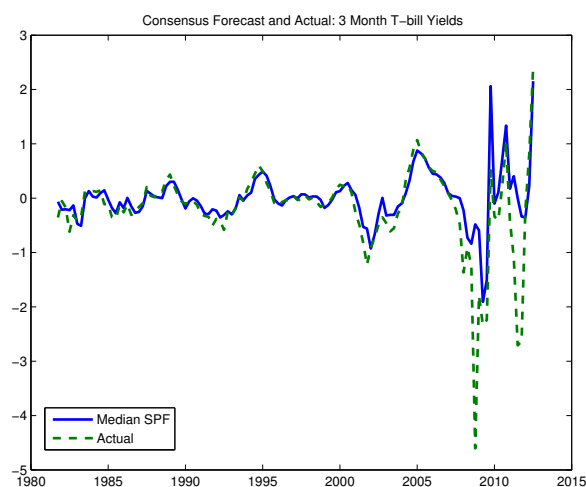
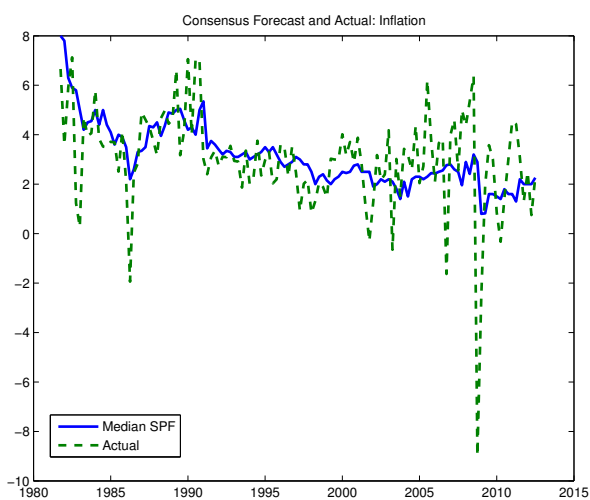
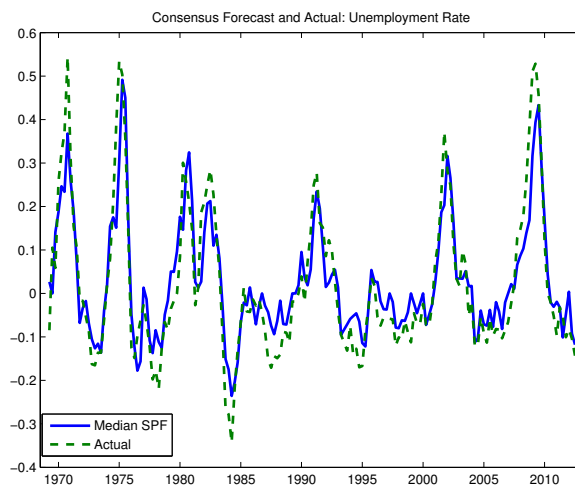
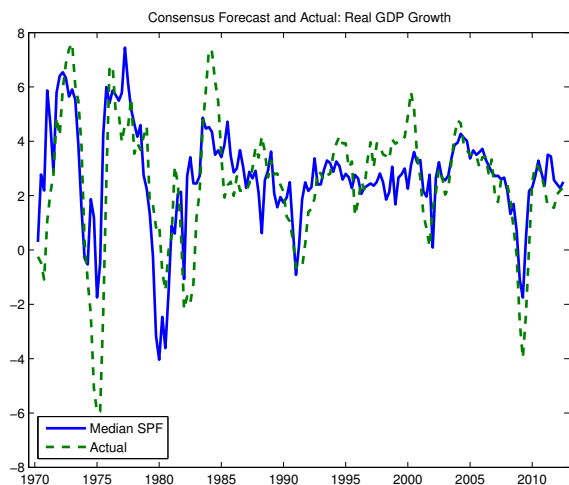
and

$$y_{jt}^{(\text{spf})} = \ln(Y_{jt}^{(\text{spf})}/Y_{j,t-1})$$

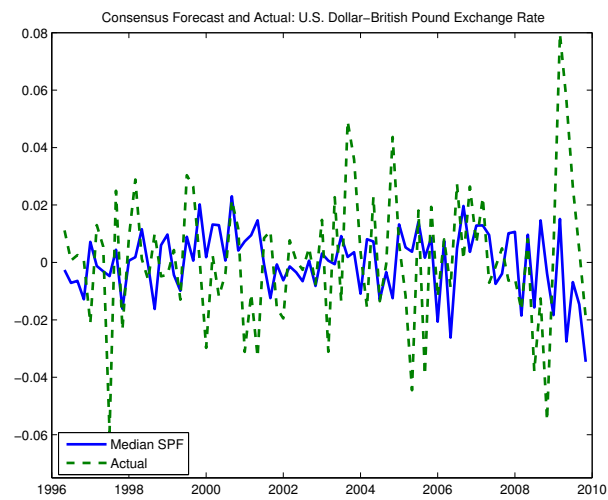
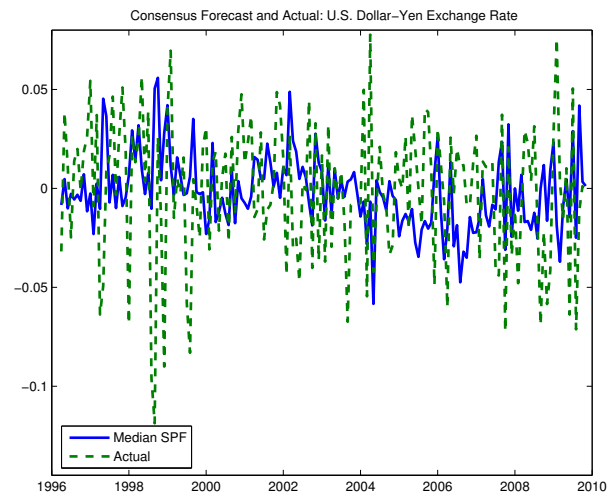
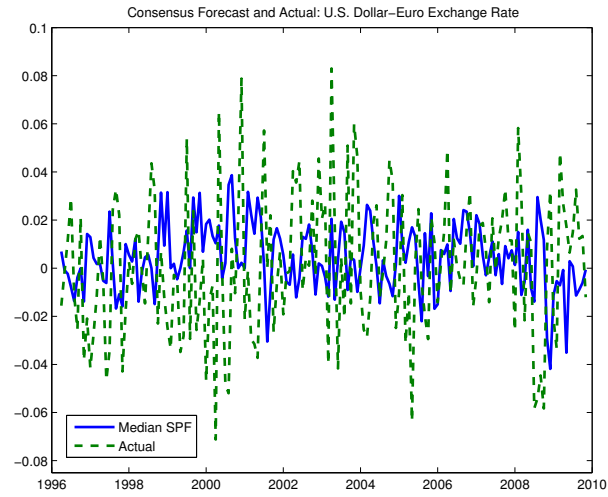
where we use the same notation as for the quarterly data. The survey data for the U.S. dollar-British pound exchange rates are only available every other month.

The economic variables that we use to explain the exchange rate are the differentials in CPI inflation and the differentials in the one month interbank interest rates. For the interest rates we use the LIBOR rate, the dollar LIBOR rate, the yen LIBOR rate, and the EURIBOR rate. These series are provided by the St. Louis Fed's FRED database.

There are two issues concerning our data that need to be kept in mind. First, for some series the data sample is relatively short. Second, we are assessing adaptive learning on aggregate data while learning takes place on an individual level. However, in both surveys the number of responses is fluctuating substantially. For example, the number of respondents for GDP in the SPF survey ranges from 9 in 1990Q2 to 131 in 1970Q4. More details



Note: The figures display the median one quarter ahead forecasts from the survey of professional forecasters as the solid line and the actual outcome of each variable as the dashed line. The variables are: real GDP growth as the (1,1) element of the matrix, the growth rate of the unemployment rate as the (1,2) element, the inflation rate (2,1), the growth rate of the three month t-bill rate (2,2), the growth rate of the ten year government bond yields (3,1), and the average AAA corporate bond yields (3,2).



Note: The figures display the median one month ahead forecasts from a survey among forecasters collected by Consensus Economics of London as the solid line and the actual outcome of each variable as the dashed line. The variables are top to bottom: the euro-dollar exchange rate, the yen-dollar exchange rate, and the pound-dollar exchange rate.

can be found in Appendix A.

Additionally, some respondents left the panel and were replaced by different individuals in the same institution so that the same survey identifier can correspond to different individuals. The individual responses series are, therefore, too short and unreliable to test our specifications. The aggregation of the data allows us to use the entire sample. Also, most learning models are specified in aggregate terms and it is therefore useful to know the properties of aggregated learning.

Figures 1–2 plot the series and the median survey expectations. The solid lines correspond to the median expectations and the dotted lines represent the realized data. While the survey expectations do a good job predicting the actual series, it is notable that, for all series, the actual data are substantially more volatile. This is particularly so for the inflation series and for the three exchange rates.

4 Results

4.1 Constant gain estimates

Table 2 reports the estimates of the constant gain parameters. The first column reports the value of gain parameter of the adaptive expectations model. The following columns report the values of gain parameters for autoregressive specifications and for models including both autoregressive components and economic variables. Finally, the last column reports the gain parameter of the model containing only economic predictors.

The estimates for the adaptive learning parameter vary considerably between the series. The values of the gain parameters of the autoregressive and economic models are small for most series, with 40 out of 60 figures equal to 0.001. These values are somewhat lower than the estimates obtained by Branch and Evans (2006), Milani (2007), and Orphanides and Williams (2005), which are around 0.02. However, the direct comparison of the gain values is difficult since each of the above mentioned studies uses different models and, importantly, macroeconomic data rather than SPF forecasts of the data.

Table 2: Estimates of the gain parameter, γ

Variable	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	0.086	0.004	0.004	0.004	0.004	0.002	0.003	0.091
3 month yields	0.887	0.003	0.001	0.016	0.001	0.001	0.001	0.092
10 year yields	0.512	0.001	0.001	0.001	0.001	0.018	0.001	0.001
Corp bond yields	0.886	0.051	0.001	0.026	0.001	0.001	0.001	0.001
GDP growth	0.053	0.001	0.002	0.002	0.023	0.001	0.011	0.012
Unemployment	1.000	0.003	0.040	0.002	0.001	0.002	0.001	0.001
EUR/USD	0.079	0.001	0.001	0.001	0.001	0.001	0.001	0.001
JPY/USD	0.070	0.001	0.001	0.001	0.001	0.001	0.001	0.001
UKP/USD	0.040	0.002	0.001	0.001	0.001	0.001	0.001	0.001

Note: The table reports estimates of γ obtained using the training sample. AE stands for adaptive expectations, AR(1)–AR(4) are autoregressive specifications, and an X indicates the inclusion of economic predictors, which are reported in Table 1.

The largest values of the gain have been estimated for the economic models of inflation and three month yields, which are 0.091 and 0.092, respectively. The average estimated gain across all the variables and specifications with regressors equals 0.007.⁹ These values suggest that agents use a long history of observations to make their forecasts. For quarterly data, $\gamma = 0.001$ indicates that the observation from four quarters ago receives a weight 0.996 and the weight 0.961 if it is 10 years old. The gain value of 0.091 for the economic model of inflation implies that the observation from 4 quarters ago is currently attributed the weight 0.683 and weight 0.022 if it is 10 years old.

4.2 Forecasting performance of the models

Table 3 reports the MSFE of the out-of-sample forecasts. The estimated gain parameters reported in Table 2 are used to generate these forecasts. The top panel of the table compares the performance of constant gain learning specifications to the random walk forecast, $\hat{y}_{T+1} = y_T$. The lower panel of Table 3 shows the MSFE results for the decreasing gain learning specifications in addition to the differenced random walk forecast, $\hat{y}_{T+1} = 0$. In order to make the comparison easier, the lowest MSFE for each variable are

⁹In the computation of the gain, we exclude the adaptive expectations parameter value to allow for comparison of the estimated values with previous adaptive learning studies.

Table 3: MSFE for constant and decreasing gain learning

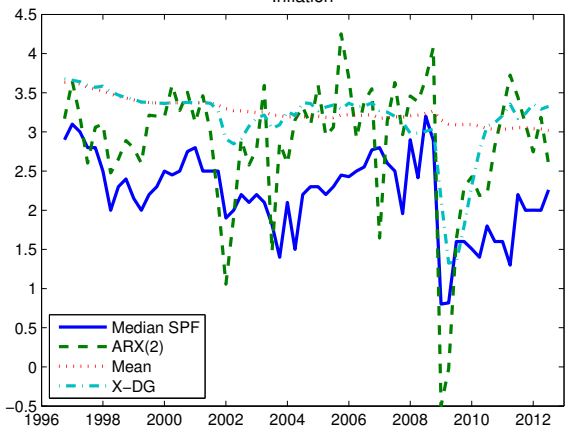
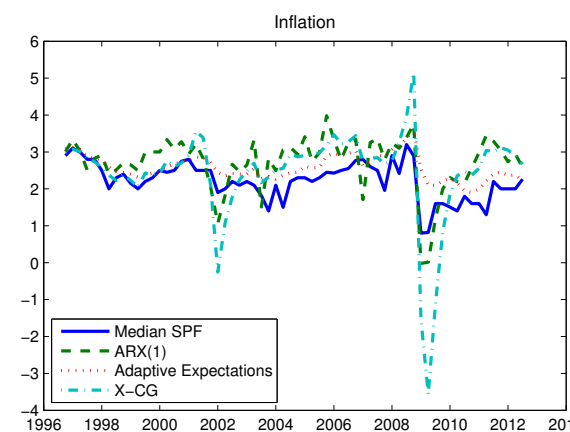
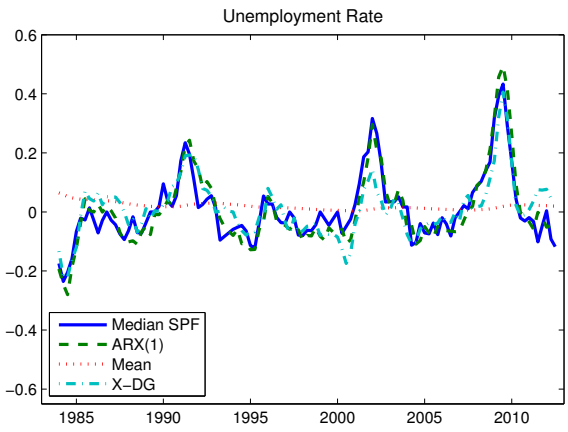
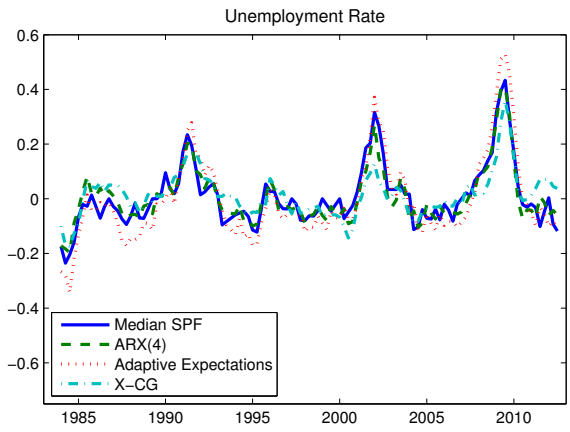
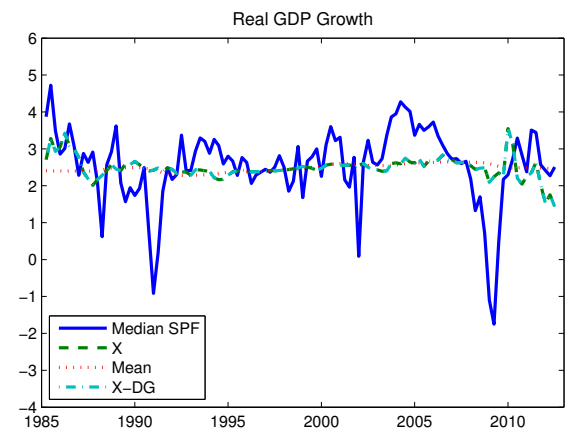
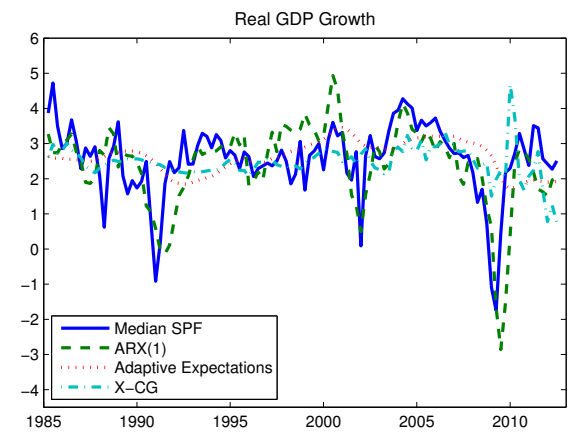
Constant Gain Least Squares									
	RW	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	3.989	0.239	0.607	0.599	0.687	0.613	0.997	0.999	1.027
3 month yields	0.878	0.815	0.785	0.675	1.135	0.743	0.729	0.698	0.793
10 year yields	0.100	0.075	0.060	0.061	0.065	0.068	0.048	0.044	0.070
Corp bond yields	1.015	0.931	0.826	0.792	0.871	0.807	0.532	0.604	1.080
GDP growth	1.575	1.143	1.097	0.983	1.087	1.217	1.164	1.168	1.018
Unemployment	0.387	0.387	0.218	0.393	0.222	0.197	0.186	0.176	0.475
EUR/USD	0.141	0.022	0.023	0.032	0.023	0.032	0.028	0.040	0.032
JPY/USD	0.220	0.039	0.035	0.043	0.036	0.044	0.040	0.048	0.046
UKP/USD	0.121	0.027	0.022	0.035	0.024	0.036	0.027	0.042	0.038
Decreasing Gain Least Squares									
	Δ RW	Mean	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	5.180	1.264	0.872	0.875	0.890	0.858	1.221	1.273	1.064
3 month yields	0.399	0.429	0.786	0.696	0.925	0.795	0.858	0.768	0.408
10 year yields	0.062	0.061	0.068	0.069	0.074	0.077	0.046	0.047	0.072
Corp bond yields	0.755	0.904	0.781	1.020	0.879	1.058	0.659	0.783	1.360
GDP growth	7.571	1.079	1.296	1.179	1.265	1.110	1.323	1.161	0.987
Unemployment	1.275	1.384	0.255	0.215	0.283	0.236	0.241	0.214	0.459
EUR/USD	0.020	0.022	0.024	0.034	0.024	0.034	0.030	0.043	0.034
JPY/USD	0.037	0.037	0.035	0.044	0.037	0.045	0.042	0.051	0.047
UKP/USD	0.025	0.026	0.022	0.036	0.024	0.037	0.027	0.045	0.039

Note: The table reports the MSFE for constant and decreasing gain learning models, where the MSFE is defined in (6). AE stands for adaptive expectations, AR(1)-AR(4) are autoregressive specifications. The column ‘Mean’ reports the mean forecast, $\hat{y}_{T+1} = T^{-1} \sum_{t=1}^T y_t$. X indicates the inclusion of economic predictors, which are reported in Table 1. The smallest MSFE for each variable is reported in bold font.

reported in bold font.

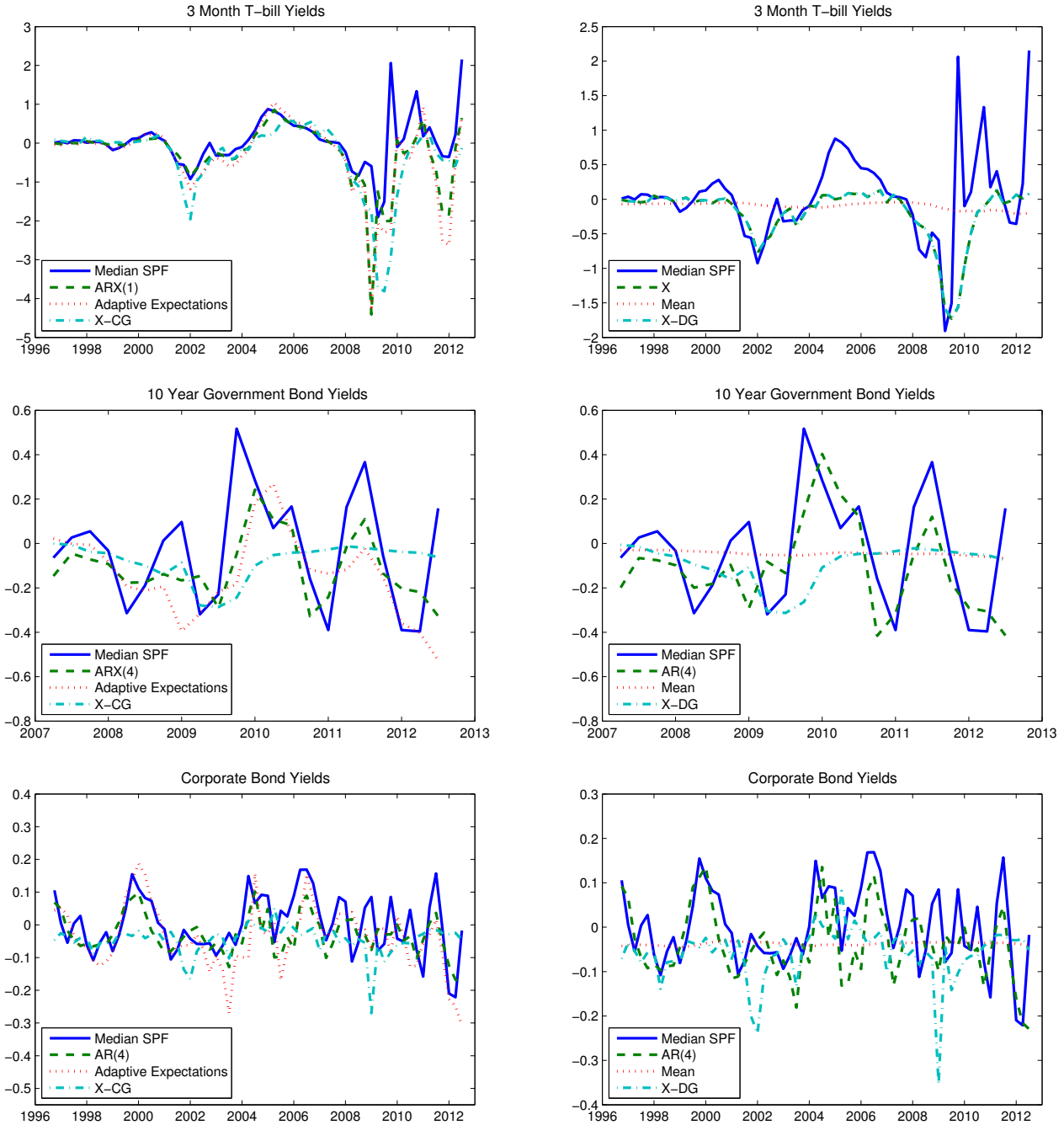
The corresponding forecasts are plotted in Figures 3–5. The left panels plot forecasts for constant gain updating while the right panels show the decreasing gain learning. Green, dashed lines represent the forecasts generated by the best CGLS model in the plots on the left and the best DGLS model in the plots on the right; red, dotted lines those by constant gain adaptive expectations in the plots on the left side and the DGLS intercept only model in the plots on the right side. Light blue dashed lines are forecasts produced by economic models and the actual median forecast is the solid, blue line.

When interpreting the forecasts, first, notice that the random walk spec-



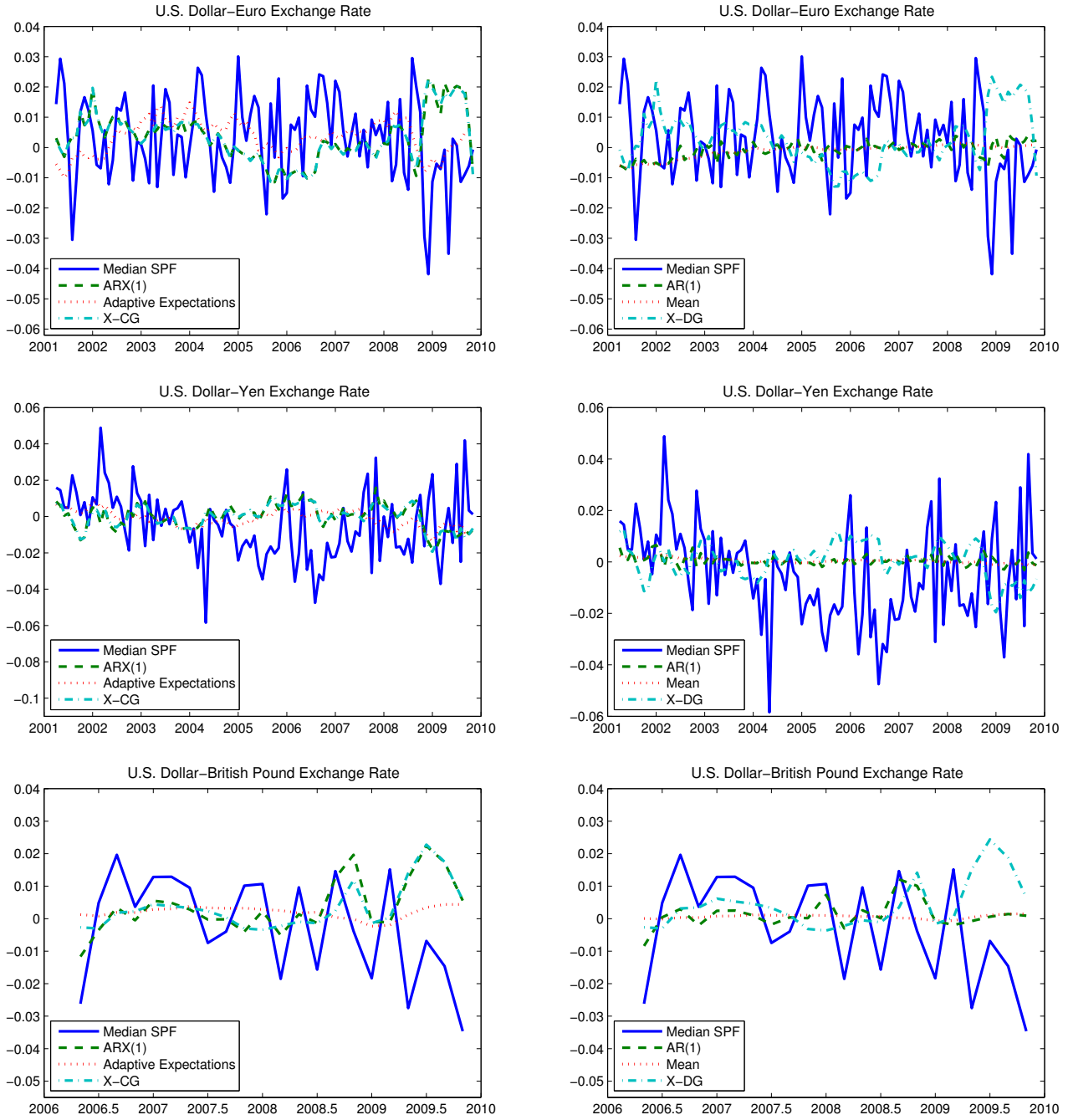
Note: The figures display the median one quarter ahead forecasts from the survey of professional forecasters as the solid line together with the forecasts of selected model: The left panels plot forecasts for constant gain updating while the right panels show the decreasing gain learning. Green, dashed lines represent the forecasts generated by the best CGLS model in the plots on the left and the best DGLS model in the plots on the right; red, dotted lines those by constant gain adaptive expectations in the plots on the left side and the DGLS intercept only model in the plots on the right side; light blue dashed lines are forecasts produced by economic models. The variables per row, top to bottom: real GDP growth, unemployment growth, and the inflation rate.

Figure 4: Median SPF forecasts and model forecasts



Note: The variables per row top to bottom are: the yields of three month U.S. t-bills, U.S. ten year government bonds, and AAA corporate bonds. For the other details see the footnote of Figure 3.

Figure 5: Median SPF forecasts and model forecasts



Note: The variables per row top to bottom are: the euro-dollar exchange rate, the yen-dollar exchange rate, and the British pound-dollar exchange rate. For the other details see the footnote of Figure 3.

ification, $\hat{y}_{T+1} = y_T$, delivers better forecasts than the differenced random walk or no-change specification, $\hat{y}_{T+1} = 0$, for the macroeconomic variables: inflation, GDP growth, and unemployment. The reverse is true for the financial time series, yields and exchange rates, for which the differenced random walk is a better benchmark. Our comparison to the random walk will therefore focus on the respectively better specification.

Table 4 reports Diebold-Mariano test statistics for forecast accuracy against the random walk forecast, $\hat{y}_{T+1} = y_T$, where significant differences are in bold font. The top panel of the table shows the results for the CGLS and the lower panel for DGLS. The positive, significant entries indicate that the alternative specification forecasts are significantly better than the random walk model. This table is therefore most relevant for the macroeconomic variables: inflation, GDP growth and unemployment. The Diebold-Mariano test statistics for forecast accuracy against the differenced random walk or no-change forecast, $\hat{y}_{T+1} = 0$, are reported in Table 5. Again, significant differences are in bold font. This table is most relevant for the interest rate and exchange rate forecasts.

The first line of Table 3 shows that adaptive expectation delivers far and away the best forecast for inflation, which is in line with the findings of Stock and Watson (2007). The left bottom panel of Figure 3 shows that adaptive expectations forecast for inflation remains closest to the SPF forecast but is also substantially less volatile than the other forecasts. For this reason it does particularly well after unstable periods, such as the recent financial crisis, where the other models predict longer or more extreme deviations from the mean than the median professional forecaster (solid blue line). The Diebold-Mariano test statistics reported in Table 4 show that adaptive expectation but also several other models beat the random walk significantly.

For the three month T-bill rates, the no-change forecast, ΔRW , cannot be beaten by any model with either constant or decreasing gains. In contrast, for the ten year bond and the corporate bond yields, many models deliver better predictions with the best forecasts originating from the constant gain ARX(4) for 10 year bond yields and AR(4) for corporate bond yields. However, the Diebold-Mariano test statistics in Table 5 show that

the improvements for the forecasts of the ten year and corporate bonds are not significant.

The results for macroeconomic variables, GDP growth and unemployment, show that the random walk has a higher MSFE than a number of models with both, constant and decreasing gains. The best models in both cases use constant gains and are the ARX(1) and ARX(4), respectively. The Diebold-Mariano test statistics in Table 4 show that these improvements are significant in both cases.

Exchange rates are known to be difficult to forecast and the result that the no-change forecast for the euro-U.S. dollar is closest to the SPF forecast implies that professional forecasters seem to confirm this. Furthermore, except for forecast from adaptive expectations all other forecasts are significantly worse than the no-change prediction, as can be seen in Table 5. In contrast, the SPF forecasts of the yen-U.S. dollar exchange rate are best matched by the AR(1) forecasts and as Table 5 shows, this forecast is significantly better than the no-change prediction. For the pound-U.S. dollar forecasts the AR(1) is again the best forecast but in this case the result is not significant.

The fact that the fundamentals-based models cannot beat the random walk in the out-of-sample forecasting exercise has been largely documented in the empirical literature on exchange rates. See Cheung, Chinn, and Pascual (2005) for the literature overview and recent findings on exchange rate predictability. Interestingly, we find here that the surveys of professional forecasters are also well approximated by random walk behavior for the euro-U.S. dollar exchange rate but for the yen and pound U.S. dollar exchange rates AR(1) models do better.¹⁰

4.3 Importance of economic variables

An interesting question is whether the economic variables add explanatory power to the purely autoregressive models. From the last column of Table 3 it is clear that models without any autoregressive component do not perform

¹⁰Remember that in this exercise, we forecast survey based expectations, in contrast to the actual data forecasts usually studied and described in Cheung, Chinn, and Pascual (2005) for instance.

Table 4: Test for predictive accuracy against random walk forecast, $\hat{y}_{T+1} = y_T$

Constant Gain Least Squares								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	2.529	2.220	2.236	2.165	2.225	1.950	1.924	2.026
3 month yields	1.289	0.802	2.482	-0.770	2.112	0.596	0.822	0.281
10 year yields	1.785	1.893	1.649	1.655	1.339	2.338	2.200	0.817
Corp bond yields	3.005	2.169	1.196	1.569	1.191	2.798	2.266	-0.220
GDP growth	1.148	2.837	4.476	2.562	2.633	1.669	2.693	1.612
Unemployment	–	6.557	-0.116	4.229	3.783	4.846	4.341	-1.065
EUR/USD	7.413	7.272	6.601	7.234	6.546	6.871	5.952	6.571
JPY/USD	6.972	6.925	6.638	6.902	6.638	6.845	6.552	6.683
UKP/USD	1.831	1.918	1.814	1.819	1.706	1.745	1.557	1.780
Decreasing Gain Least Squares								
	Mean	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	1.860	2.046	2.091	2.030	2.083	1.794	1.724	1.921
3 month yields	1.260	0.816	2.295	-0.251	1.075	0.063	0.412	1.623
10 year yields	1.077	2.032	1.558	1.661	1.177	2.286	2.278	0.767
Corp bond yields	0.443	2.385	-0.025	1.466	-0.252	2.388	1.300	-0.933
GDP growth	1.333	4.067	3.828	1.972	2.969	1.478	2.634	1.648
Unemployment	-4.106	7.058	5.735	3.024	3.832	3.866	4.371	-0.897
EUR/USD	7.206	7.232	6.512	7.188	6.453	6.779	5.772	6.479
JPY/USD	6.963	6.927	6.620	6.898	6.609	6.782	6.457	6.660
UKP/USD	1.856	1.927	1.800	1.809	1.666	1.725	1.473	1.768

Note: The table reports the Diebold-Mariano (1995) test statistic for predictive accuracy against the forecast $\hat{y}_{T+1} = y_T$. The loss function is $e_{RW}^2 - \hat{e}_i^2$, where i stands for the different models in the table. For the models see Table 3. Bold font indicates significance at the 5% level.

well in replications SPF forecasts. However, for the ten year yields, GDP growth and the unemployment rate, the best models contain economic variables in addition to autoregressive components. Furthermore, for the three month t-bill yields models with economic variables generally do better than the corresponding models without economic variables, even if no model can beat the no-change forecast. This is in stark contrast to the exchange rates where for each model the purely autoregressive version performs better than the same model with economic variables.

Table 5: Test for predictive accuracy against random walk forecast, $\hat{y}_{T+1} = 0$

Constant Gain Least Squares								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	16.301	14.423	14.437	13.519	14.270	9.807	9.201	8.192
3 month yields	-1.269	-1.033	-0.900	-1.213	-0.999	-1.106	-1.148	-1.202
10 year yields	-0.443	0.083	0.048	-0.171	-0.285	0.646	1.145	-0.525
Corp bond yields	-0.738	-0.364	-0.234	-0.605	-0.340	1.639	1.009	-1.612
GDP growth	12.434	15.428	14.220	15.730	13.467	16.525	14.139	13.646
Unemployment	3.479	3.977	3.982	3.941	4.069	4.067	4.132	3.256
EUR/USD	-0.655	-3.947	-3.455	-3.801	-3.362	-4.520	-4.907	-3.459
JPY/USD	-1.130	3.049	-2.190	0.945	-2.376	-1.376	-3.212	-3.074
UKP/USD	-1.103	1.500	-1.340	0.295	-1.659	-0.443	-1.862	-1.859
Decreasing Gain Least Squares								
	Mean	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	10.083	13.197	12.929	12.997	13.059	9.032	7.619	12.168
3 month yields	-1.209	-1.044	-0.930	-1.119	-1.046	-1.282	-1.292	-0.058
10 year yields	0.109	-0.262	-0.288	-0.482	-0.588	0.757	0.757	-0.589
Corp bond yields	-1.801	-0.143	-1.170	-0.649	-1.363	0.548	-0.137	-1.945
GDP growth	13.059	12.844	12.743	14.159	14.144	14.104	14.224	13.538
Unemployment	-1.947	3.873	4.026	3.764	3.931	3.919	4.011	3.173
EUR/USD	-2.233	-4.110	-3.751	-3.971	-3.602	-4.589	-5.147	-3.673
JPY/USD	0.463	2.820	-2.343	0.456	-2.615	-2.073	-3.623	-3.097
UKP/USD	-0.969	1.556	-1.300	0.250	-1.642	-0.480	-1.899	-1.845

Note: The table reports the Diebold-Mariano (1995) test statistic for predictive accuracy against the forecast $\hat{y}_{T+1} = 0$. The loss function is $e_{\Delta RW}^2 - \hat{e}_i^2$, where i stands for the different models in the table. For the models see Table 3. Bold font indicates significance at the 5% level.

4.4 Constant gain least squares versus alternative specifications

A further, interesting comparison is that of CGLS forecasts against DGLS forecasts. Table 3 shows that, with the exceptions of the series where a no-change forecast cannot be beaten, CGLS provides the model with the lowest MSFE. This comparison is more formally addressed in the upper panel of Table 6, which reports Diebold-Mariano test statistic for the predictive accuracy of each model estimated with CGLS against the same model estimated with DGLS. A negative entry indicates a smaller MSFE of the CGLS and a bold entry a significant difference. It can be seen that CGLS generates

Table 6: Test for predictive accuracy: CGSL against DGLS and constant parameter modes

CGSL against DGLS								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	-10.960	-8.175	-6.941	-4.643	-7.915	-2.417	-2.914	-0.105
3 month yields	1.198	-0.371	-1.001	1.365	-1.306	-1.312	-0.900	1.856
10 year yields	0.526	-1.401	-1.785	-1.439	-1.807	0.989	-0.658	-1.143
Corp bond yields	0.114	0.909	-2.920	-0.217	-3.163	-2.660	-2.617	-2.450
GDP growth	1.058	-1.779	-3.278	-2.978	2.339	-1.455	0.360	0.488
Unemployment	-4.106	-4.813	3.182	-4.694	-2.048	-4.739	-2.184	0.906
EUR/USD	-0.318	-4.104	-3.381	-4.086	-3.461	-4.818	-5.509	-2.694
JPY/USD	1.422	-0.256	-2.451	-2.445	-3.377	-3.773	-4.413	-2.016
UKP/USD	1.154	1.552	-0.839	0.019	-1.068	-0.591	-1.539	-1.590
CGSL against constant parameter models								
	Mean	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	-12.249	-9.772	-7.956	-8.071	-7.429	-4.814	-2.881	-3.556
3 month yields	1.246	1.132	1.440	1.278	1.543	1.308	1.753	1.595
10 year yields	0.504	-1.254	-0.466	-1.540	-0.543	1.259	0.275	-0.191
Corp bond yields	-0.032	0.756	-1.631	-0.973	-2.295	-2.762	-2.331	-2.333
GDP growth	0.809	-1.561	-3.731	-2.920	0.230	-1.495	-0.321	-2.414
Unemployment	-6.288	-1.304	2.895	-3.190	-1.284	-2.516	-1.339	-0.808
EUR/USD	-2.238	-2.664	-4.050	-4.119	-3.829	-5.987	-3.608	-4.516
JPY/USD	-0.432	-5.993	-5.549	-5.096	-6.018	-5.230	-5.122	-5.418
UKP/USD	1.104	0.943	1.078	1.015	0.939	-0.842	-1.028	1.595

Note: The table reports the Diebold-Mariano (1995) test statistic for predictive accuracy of the CGSL models against the forecasts of the DGLS and constant parameter specifications for the loss functions $e_{\text{CGSL},i}^2 - e_{\text{DGLS},i}^2$ and $e_{\text{CGSL},i}^2 - e_{\text{const. para.},i}^2$, where i stands for the different models in the table. For the models see Table 3. Bold font indicates significance at the 5% level.

significantly more accurate forecasts than DGLS in 33 out of 72 cases.¹¹ In contrast, DGLS has a significantly lower MSFE compared to CGSL only in two instances.

Finally, we consider the question whether the time variation in the parameters is important enough to outweigh the additional noise introduced by the repeated estimation. For this purpose, we compare the CGSL forecasts to those with constant parameters, where the parameters are estimated with the sample ending at T_{init} . The Diebold-Mariano test statistics are re-

¹¹Branch and Evans (2006) also find that a CGSL VAR(1) performs best when predicting GDP growth and inflation, using SPF.

ported in the lower panel of Table 6, where again a negative entry indicates a smaller MSFE of the CGLS and a bold entry a significant difference. Here, in 33 out of 72 entries, CGLS is a significantly better predictor than the constant parameters model. The latter beats the CGLS only once, for ARX(1) unemployment, which itself is not the best CGLS model to predict SPF forecasts of unemployment and was outperformed by the ARX(4) specification. These results suggest that time varying parameters are important for matching SPF forecasts and that CGLS is the best approach out of the models considered here.

5 Robustness checks

In this section, we investigate in how far the results of the previous section depend on the choice of the admissible range of the constant gain parameter, the choice of initialization of the parameters in CGLS, and the choice of T_{init} and therefore, implicitly, the choice of the out-of-sample period. We also compare the forecasting performance of the DGLS using rolling and expanding windows.

5.1 Gain parameter estimation

In the baseline estimation, we restricted the constant gain parameter to $\gamma \in (0, 1]$, which is the common range in the literature. Now, we repeat the estimation and forecasting process while allowing γ to be negative, that is, we minimize (5) with $\gamma \in [-1, 1]$. All the other parameters and settings of the estimation remain unchanged.

The results of this exercise are reported in Appendix B in Table 7. The upper panel of the table reports the estimated gain values for each variable. Nine out of 63 fitted gain parameter values are negative. The second panel reports the MSFE for the CGLS forecasts where entries in bold font correspond to negative γ 's. Comparing these MSFE values to those obtained with positive γ 's in Table 3 shows that for seven out of nine cases the MSFE with negative γ is larger, and for four substantially larger, whereas only in two cases the MSFE is mildly improved.

The two lower panels display the Diebold-Mariano test statistic for the MSFE of the CGLS against the DGLS and constant parameter specifications. The positive entries indicate that the CGLS generates less accurate forecasts than the alternative model. Again, entries in bold correspond with negative constant gains. In 16 out of 18 cases, the alternative specifications beats CGLS and in 11 cases significantly so. In contrast, the same models with positive constant gain parameters CGLS provide significantly better forecasts than the alternatives in 8 cases as can be seen in Table 6. The results suggest therefore that the originally imposed restriction to positive gain parameter values is a sensible one.

5.2 Initialization of CGLS

An important aspect of CGLS is the initialization.¹² Diffuse priors have the advantage of imposing little knowledge and thus exerting little influence on the subsequent parameter estimates. However, it is interesting to check the robustness of this choice against another frequently used method, namely estimating $\theta_0^{(i)}$ by OLS using a small initial sample. Here we use the first 20 observations. Again all other choices remain the same as in the baseline estimation and forecasting, including the restriction to positive gain parameters.

Table 8 in Appendix B displays the results. The upper panel shows fitted gain parameter values. They are slightly higher than the ones obtained in the main estimation and reported in Table 2. In fact the average gain is 0.015 in contrast to 0.007 in the baseline exercise. The second panel shows the resulting MSFEs, and the last two panels display Diebold-Mariano statistics comparing CGLS to DGLS and the constant parameter forecast. The results are qualitatively similar to the ones obtained in the baseline estimation and forecasting exercise. Generally, the MSFE are similar, yet slightly larger than those from the diffuse prior reported in Table 3. The significance of the DM test statistics is also similar. However, for one specification, the yen-U.S. dollar exchange rate DGLS now delivers a significantly better forecast than CGLS, which was not the case with the diffuse prior. Given

¹²See Carceles-Poveda and Giannitsarou (2007) and Berardi and Galimberti (2012b) for reviews of the initialization of CGLS algorithms.

the similar, yet slightly better forecasts from diffuse priors, we favor the diffuse prior initialization procedure.

5.3 OLS with rolling windows

Given that decreasing gain least squares is equivalent to OLS with expanding window when initializing using OLS in the training sample, it is of interest to compare it to forecasts from OLS from rolling windows. While expanding windows should lead to better forecasts in the absence of structural breaks, rolling window can lead to better forecasts in unstable models, see, e.g., Pesaran and Timmermann (2007) for a discussion of the optimal window size.

Table 9 in Appendix B reports the MSFE of the forecasts from rolling window of length $T_{\text{init}} = 60$ in the upper panel and the Diebold-Mariano test statistics of CGLS against rolling window forecasts in the lower panel. Table 9 demonstrates that rolling window OLS delivers the lowest MSFE only for two series: the euro and the yen-U.S. dollar exchange rate. The Diebold-Mariano test statistics in the lower panel of Table 9 show that the result is significant only for the yen-U.S. dollar exchange rate. For the remaining variables, the forecasts have a larger MSFE than CGLS and for four of them this difference is significant. Hence, although the OLS forecasts improve when they are generated with the rolling window, the CGLS still outperforms OLS algorithm. In 49 out of 72 cases the CGLS forecasts are better than DGLS and in 16 instances significantly so. Our result that the CGLS better approximates survey expectations than DGLS is thus robust to the choice of the estimation window.

5.4 Selection of the out-of-sample period

A critical choice in any forecast evaluation is the split between training sample and forecast sample, see Hansen and Timmermann (2012). As a robustness check, we repeat the forecasting exercise for a longer forecast period and set $T_{\text{init}} = 48$. All other settings are as in the main estimation and forecasting exercise.

Table 10 in Appendix B reports the estimated constant gain parameters

in the upper panel and the MSFE of the CGLS and DGLS forecasts in the two lower panels. Table 11 in Appendix B reports the Diebold-Mariano test statistics of CGLS against DGLS and against the constant parameter forecasts.

The fitted gain in the shorter estimation sample is the same as in the baseline estimation for most models and a little higher for a small subset of models. The MSFE comparison gives a qualitatively similar answer to the ones reported in the baseline forecasting exercise with two interesting exceptions. First, for inflation the AR(1) with constant gain now produces predictions as good as adaptive expectations did in the main estimation. Second, for GDP the model with economic variables now provides the best match of the SPF forecasts. Both panels of Table 11 indicate that the CGLS still produces the best forecasting models. In 47 out of 72 models, CGLS outperforms DGLS and in 29 cases the improvement is significant. Similarly 49 constant gain specifications generate better forecasts than models with constant parameters. For 43 models the MSFE difference is significant, while in the baseline exercise this was the case for 33 models. The results thus suggest a slight improvement of the constant gain algorithm when a shorter in-sample period is considered. Overall, the forecasting results are not qualitatively affected.

6 Conclusion

In this paper, we evaluate in how far adaptive learning models can replicate the expectations of professional forecasters. We relax the rational expectations informational assumptions by postulating that, instead of using economic model based forecasts, agents use simple time-series models to make predictions. This type of forecasting rule embodies very low computational cost as it is based on the past values of observed variables only. In addition to pure autoregressive specifications, we experiment with one or more economic regressors that may improve forecasts. We assume that agents update the parameters of their forecasting rules using recursive least squares.

When we fit constant gain models to the survey data, we find that relatively small gains (values smaller than 0.092) provide the best fit, which

implies that professional forecasters use rather long data samples.

The comparison between constant gain and decreasing gain specifications suggests that constant gain algorithm generates better forecasts. For seven out of nine variables, we find a model that displays lower MSFE than the random walk benchmark. In all those seven cases, constant gain specifications deliver the best predictions. Similarly, CGLS is a significantly better predictor than the constant parameters model.

We find that time-series specifications deliver more accurate predictions than models with only economic variables. Specifically, for all three macroeconomic series, the economic variables on their own are the weakest predictors. However, adding economic variables to autoregressive specifications improves predictive accuracy.

When comparing the forecasts of macroeconomic series to the financial ones, we observe that for macroeconomic variables, many models significantly beat the random walk benchmark. It appears that survey expectations are well represented by more complex specifications than the random walk. In contrast, no change prediction is much harder to beat significantly for financial variables with the notable exception of the yen-U.S. dollar exchange rate.

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Appendix A: Additional details of the data

We use two survey datasets. The first one, provided by Federal Reserve Bank of Philadelphia, is the Survey of Professional Forecasters (SPF). The second dataset is collected by Consensus Economics of London.

From the SPF, we use the following quarterly series: the U.S. CPI inflation rate, the three month U.S. T-bill rate, the ten year U.S. government bond rate, Moody's AAA corporate bond yields, U.S. real GDP, and the U.S. unemployment rate. From Consensus Economics of London, we use the following exchange rate surveys: EUR/USD and JPY/USD, which are released monthly, and GBP/USD, which is released every other month.

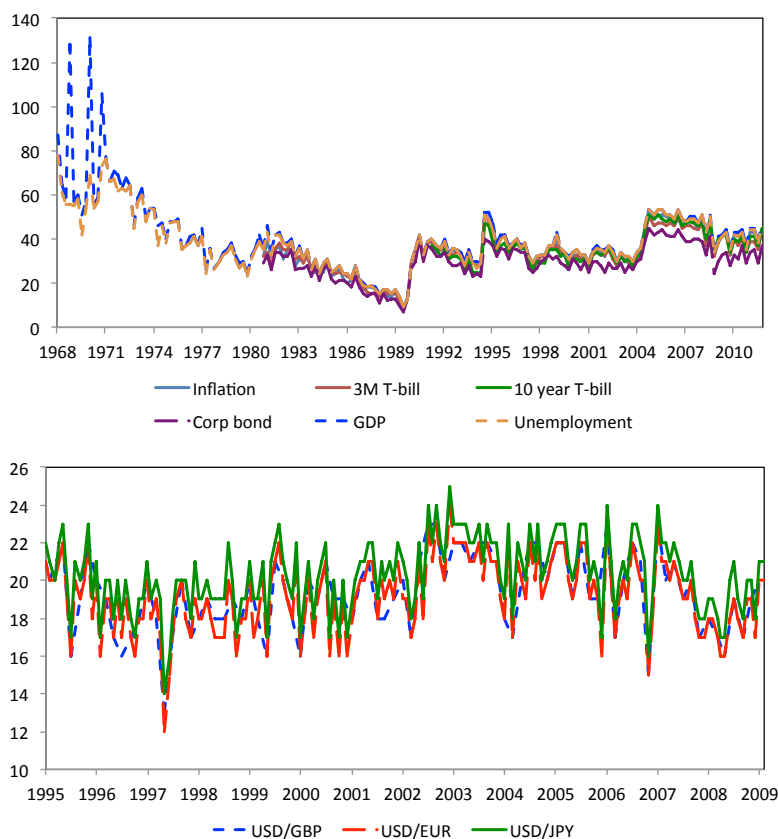
The SPF was initially conducted by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) from the fourth quarter of 1968 and was taken over by the Philadelphia Fed from the second quarter of 1990.

Both surveys use respondents among the professional forecasters. Consensus Economics of London collects results from a survey among up to 150 professional market participants and forecasting agencies for their subjective expectations on several exchange rates. The institutions of origin of the respondents include commercial and investment banks, research and forecasting institutes, and consulting firms.

The SPF classifies the survey participants into three categories: (i) financial services provider, (ii) nonfinancial services, (iii) unknown industry. The financial services category includes the following industries: Insurance, Investment Banking, Commercial Banking, Payment Services, Hedge Funds, Mutual Funds, Association of Financial Service Providers, and Asset Management. Nonfinancial services providers include: Manufacturers, Universities, Forecasting Firms, Investment Advisors, Pure Research Firms, and Consulting Firms.

In both surveys, the response rate of the individual market participants is less than 100 percent. Often, the respondents leave the panel and are replaced by others individuals. In addition, in the SPF, in some cases an individual who drops out of the panel for a large number of periods suddenly

Figure 6: The number of individual responses per period



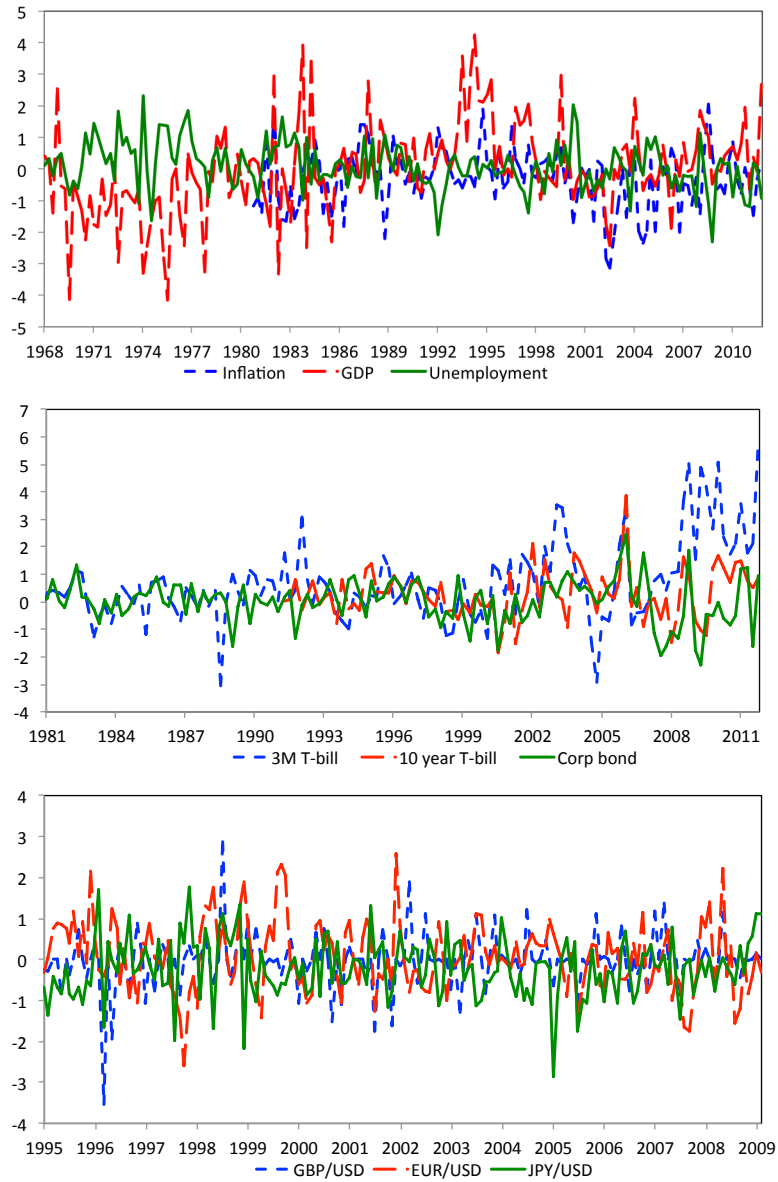
Note: The figure shows the number of respondents over time. The top panel shows the number of individual responses in the SPF series and the lower panel displays the the number of individual responses Consensus Economics of London series.

re-enters, suggesting that the same identifier might have been assigned to different forecasters. The number of responses for each series is plotted in Figure 6, where the substantial changes in the number of respondents can be seen.

In order to assess the symmetry of the response distribution, Figure 7 plots the skewness of individual responses over time. The top two panels show the skewness of the distribution of the responses in the SPF time series and the lower panel that of the time series of the individual responses collected by Consensus Economics of London. All skewness series fluctuate

around zero, which suggests that on average the distributions are symmetric. For all series, there are, however, episodes where the skewness is substantially different from zero. For example, in the 70's the GDP forecasts were usually skewed to the left suggesting outlying or extreme observations in the left tail of the distribution. The inflation forecasts, during the same period, exhibited the positive skewness indicating the presence of outliers in the right tail. In the recent years, T-bill forecasts are skewed to the right, so that there are a few large forecasts. These episodes suggest that the robustness properties of the median are useful when considering the aggregate forecasts.

Figure 7: Skewness of the distribution of individual responses



Note: The figure shows the time series of the skewness of individual responses in each period. The top panel shows the quarterly SPF time series of the individual responses' skewness and the lower panel displays the monthly time series of the skewness of individual responses collected by Consensus Economics of London.

Appendix B: Additional tables

Table 7: Results for constant gain learning with $\gamma \in [-1, 1]$

	Estimates of γ							
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	0.086	-0.004	0.004	0.004	0.004	0.002	0.003	0.091
3 month yields	0.887	0.003	0.001	0.016	0.001	0.001	0.001	0.092
10 year yields	0.512	-0.054	0.001	0.000	0.000	0.018	0.001	0.000
Corp bond yields	0.886	0.051	0.001	0.026	0.001	0.001	0.001	0.001
GDP growth	0.053	-0.010	-0.164	-0.038	0.023	0.001	0.011	-0.003
Unemployment	1.000	-0.218	0.040	0.002	0.001	0.002	0.001	-0.025
EUR/USD	0.079	0.000	0.000	0.000	0.000	0.000	0.000	0.000
JPY/USD	0.070	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UKP/USD	0.040	-0.008	0.000	0.001	0.000	0.000	0.000	0.000
MSFE: Constant Gain Least Squares								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	0.239	1.415	0.599	0.687	0.613	0.997	0.999	1.027
3 month yields	0.815	0.785	0.675	1.135	0.743	0.729	0.698	0.793
10 year yields	0.075	0.072	0.061	0.062	0.062	0.048	0.044	0.062
Corp bond yields	0.931	0.826	0.792	0.871	0.807	0.532	0.604	1.080
GDP growth	1.143	1.388	2.416	1.330	1.217	1.164	1.168	1.011
Unemployment	0.387	0.333	0.393	0.222	0.197	0.186	0.176	0.553
EUR/USD	0.022	0.020	0.020	0.020	0.020	0.020	0.020	0.020
JPY/USD	0.039	0.037	0.037	0.037	0.037	0.037	0.037	0.037
UKP/USD	0.027	0.021	0.025	0.024	0.025	0.025	0.025	0.025
Test for predictive accuracy: CGSL against DGLS								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	-10.960	8.329	-6.941	-4.643	-7.915	-2.417	-2.914	-0.105
3 month yields	1.198	-0.371	-1.001	1.365	-1.306	-1.312	-0.900	1.856
10 year yields	0.526	1.640	-1.785	-0.482	-0.588	0.989	-0.658	-0.589
Corp bond yields	0.114	0.909	-2.920	-0.217	-3.163	-2.660	-2.617	-2.450
GDP growth	1.058	3.981	4.204	3.576	2.339	-1.455	0.360	0.578
Unemployment	-4.106	2.106	3.182	-4.694	-2.048	-4.739	-2.184	2.422
EUR/USD	-0.318	-4.110	-3.751	-3.971	-3.602	-4.589	-5.147	-3.673
JPY/USD	1.422	2.820	-2.343	0.456	-2.615	-2.073	-3.623	-3.097
UKP/USD	1.154	-1.208	-1.300	0.019	-1.642	-0.480	-1.899	-1.845
Test for predictive accuracy: CGSL against constant parameter models								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	-12.249	1.266	-7.956	-8.071	-7.429	-4.814	-2.881	-3.556
3 month yields	1.246	1.132	1.440	1.278	1.543	1.308	1.753	1.595
10 year yields	0.504	1.776	-0.466	-0.448	-0.471	1.259	0.275	-0.777
Corp bond yields	-0.032	0.756	-1.631	-0.973	-2.295	-2.762	-2.331	-2.333
GDP growth	0.809	4.220	5.518	3.743	0.230	-1.495	-0.321	-3.335
Unemployment	-6.288	3.047	2.895	-3.190	-1.284	-2.516	-1.339	3.863
EUR/USD	-2.238	-4.244	-4.222	-5.047	-3.963	-7.126	-4.280	-4.363
JPY/USD	-0.432	-6.156	-5.307	-5.155	-5.973	-5.407	-5.330	-5.295
UKP/USD	1.104	0.232	-0.635	1.015	-0.470	-1.025	-1.688	-1.464

Note: For details see Table 2, 3, and 6 except that γ was optimized over the range $[-1, 1]$. In bold font

are the MSFE and DM statistics associated with negative γ .

Table 8: Results for constant gain learning with OLS initialization

	Estimates of γ							
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	0.086	0.014	0.015	0.004	0.003	0.009	0.007	0.090
3 month yields	0.887	0.004	0.041	0.015	0.011	0.002	0.002	0.092
10 year yields	0.512	0.001	0.001	0.058	0.001	0.019	0.006	0.001
Corp bond yields	0.886	0.008	0.002	0.005	0.003	0.012	0.004	0.004
GDP growth	0.053	0.009	0.015	0.094	0.043	0.005	0.004	0.001
Unemployment	1.000	0.001	0.105	0.001	0.006	0.001	0.002	0.131
EUR/USD	0.079	0.008	0.005	0.034	0.021	0.003	0.001	0.004
JPY/USD	0.070	0.001	0.001	0.001	0.001	0.001	0.001	0.002
UKP/USD	0.040	0.001	0.001	0.004	0.001	0.002	0.001	0.001
MSFE: Constant Gain Least Squares								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	0.239	0.715	0.711	0.850	0.832	1.173	1.281	1.016
3 month yields	0.815	0.807	0.805	1.124	0.874	0.844	0.761	0.793
10 year yields	0.075	0.069	0.069	0.075	0.080	0.048	0.048	0.075
Corp bond yields	0.931	0.781	1.027	0.873	1.042	0.664	0.775	1.336
GDP growth	1.143	1.336	1.233	1.699	1.340	1.345	1.189	1.013
Unemployment	0.387	0.248	0.626	0.263	0.248	0.220	0.207	0.811
EUR/USD	0.022	0.023	0.034	0.021	0.033	0.030	0.045	0.034
JPY/USD	0.039	0.037	0.046	0.038	0.047	0.043	0.052	0.047
UKP/USD	0.027	0.020	0.033	0.025	0.035	0.027	0.043	0.038
Test for predictive accuracy: CGSL against DGLS								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	-10.960	-3.184	-2.944	-3.682	-4.487	-1.056	0.120	-0.137
3 month yields	1.198	1.079	0.756	1.361	1.824	-0.990	-0.684	1.856
10 year yields	0.526	0.354	-0.289	0.336	1.711	1.097	1.152	1.859
Corp bond yields	0.114	0.084	0.873	-1.018	-1.586	0.450	-0.991	-1.525
GDP growth	1.058	1.984	1.066	1.905	2.641	2.481	2.663	1.588
Unemployment	-4.106	-5.497	3.527	-5.677	3.135	-3.928	-2.183	1.527
EUR/USD	-0.318	-2.299	0.780	-2.303	-0.658	1.993	3.927	1.349
JPY/USD	1.422	3.495	4.827	1.812	2.475	0.832	0.453	1.817
UKP/USD	1.154	-0.984	-1.621	1.874	-1.040	-0.415	-0.666	-1.646
CGLS against constant parameter models								
	AE	AR(1)	AR(1)SM	AR(2)	AR(2)SM	AR(4)	AR(4)SM	SM
Inflation	-12.249	-8.443	-5.505	-7.409	-6.372	-4.153	-0.889	-3.640
3 month yields	1.246	1.155	1.801	1.274	1.576	1.410	1.743	1.595
10 year yields	0.504	1.518	0.658	0.556	0.950	1.285	1.046	0.373
Corp bond yields	-0.032	-0.066	-1.229	-1.703	-1.749	-0.076	-1.697	-2.154
GDP growth	0.809	2.141	-2.053	1.934	1.004	1.412	-0.082	-3.223
Unemployment	-6.288	4.433	3.478	-0.434	1.285	-0.420	0.340	1.347
EUR/USD	-2.238	-2.267	-3.212	-3.589	-3.047	-5.290	-3.033	-4.026
JPY/USD	-0.432	-5.860	-5.274	-4.839	-5.832	-4.961	-4.909	-5.302
UKP/USD	1.104	-0.055	0.883	1.270	1.059	-0.872	-1.098	1.601

Note: For details see Table 2, 3, and 6 except that the parameters were initialized using OLS over the first 20 observations. In bold font are the lowest MSFE forecasts per variables, and the significant DM test statistics at the 5% significance level.

Table 9: Results for decreasing gain learning over rolling windows

	MSFE: Decreasing Gain Least Squares							
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	0.698	0.730	0.669	0.907	0.636	1.486	1.238	0.658
3 month yields	0.461	0.882	0.749	1.185	0.930	1.338	1.156	0.524
10 year yields	0.062	0.068	0.069	0.074	0.077	0.046	0.049	0.070
Corp bond yields	0.890	0.765	0.875	0.914	0.998	0.717	0.803	1.141
GDP growth	1.252	1.419	1.229	1.359	1.187	1.422	1.241	1.236
Unemployment	1.511	0.321	0.331	0.324	0.327	0.311	0.321	0.662
EUR/USD	0.020	0.021	0.033	0.022	0.032	0.029	0.044	0.032
JPY/USD	0.037	0.028	0.037	0.031	0.039	0.038	0.049	0.046
UKP/USD	0.026	0.023	0.043	0.027	0.046	0.029	0.051	0.043
Test for predictive accuracy: CGSL against DGLS								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	ARX(4)	X
Inflation	-7.229	-0.520	-0.758	-0.703	-0.231	-1.021	-1.323	1.133
3 month yields	1.099	-1.364	-0.624	-1.038	-1.903	-1.077	-1.174	2.381
10 year yields	0.509	-1.408	-1.647	-1.348	-1.516	0.864	-0.709	0.190
Corp bond yields	0.175	1.367	-0.876	-1.165	-1.697	-2.616	-2.063	-0.939
GDP growth	-1.265	-1.820	-2.481	-2.478	1.051	-1.693	-1.850	-2.857
Unemployment	-4.147	-4.478	3.156	-4.211	-2.775	-5.087	-3.209	-2.605
EUR/USD	0.747	1.700	-0.409	1.083	0.288	-0.094	-1.280	0.283
JPY/USD	2.063	3.828	2.703	1.942	1.581	0.386	-0.182	0.155
UKP/USD	1.021	-1.281	-2.193	-1.606	-2.041	-0.991	-1.681	-2.009

Note: For details see Tables 3 and 6. In bold font are the forecasts with the smallest MSFE in the top panel, and significant DM test statistics at the 5% critical value.

Table 10: Results for an alternative forecast sample

	Estimates of the gain parameter, γ								
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	AR(4)X	X	
Inflation	0.344	0.004	0.005	0.004	0.005	0.003	0.003	0.080	
3 month yields	0.817	0.004	0.001	0.003	0.015	0.002	0.001	0.062	
10 year yields	0.611	0.001	0.001	0.001	0.001	0.001	0.001	0.001	
Corp bond yields	0.301	0.298	0.012	0.034	0.016	0.001	0.001	0.016	
GDP growth	1.000	0.306	0.008	0.003	0.011	0.060	0.012	0.002	
Unemployment	1.000	0.010	0.007	0.003	0.004	0.002	0.002	0.015	
EUR/USD	0.093	0.001	0.001	0.001	0.001	0.001	0.001	0.116	
JPY/USD	0.065	0.001	0.001	0.001	0.001	0.001	0.001	0.001	
UKP/USD	0.052	0.001	0.038	0.001	0.001	0.001	0.001	0.052	
MSFE: Constant Gain Least Squares									
	RW	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	AR(4)X	X
Inflation	3.478	0.553	0.482	0.498	0.536	0.502	0.803	0.770	0.788
3 month yields	0.740	0.660	0.672	0.563	0.771	0.752	0.672	0.571	0.594
10 year yields	0.071	0.055	0.044	0.046	0.049	0.052	0.034	0.036	0.054
Corp bond yields	0.902	0.800	1.050	0.798	0.799	0.848	0.501	0.552	1.152
GDP growth	2.480	2.480	3.325	1.525	1.530	1.556	2.218	1.624	1.391
Unemployment	0.408	0.408	0.277	0.242	0.233	0.221	0.188	0.182	0.487
EUR/USD	0.159	0.028	0.026	0.036	0.027	0.037	0.030	0.044	0.058
JPY/USD	0.208	0.036	0.033	0.042	0.035	0.043	0.038	0.046	0.044
UKP/USD	0.111	0.022	0.018	0.034	0.018	0.025	0.021	0.030	0.034
MSFE: Decreasing Gain Least Squares									
	Δ RW	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	AR(4)X	X
Inflation	5.938	1.127	0.762	0.772	0.787	0.765	1.070	1.113	0.978
3 month yields	0.347	0.378	0.664	0.588	0.781	0.671	0.724	0.648	0.352
10 year yields	0.048	0.052	0.051	0.053	0.058	0.061	0.039	0.040	0.057
Corp bond yields	0.832	0.987	0.696	0.897	0.788	0.931	0.626	0.734	1.331
GDP growth	8.128	1.237	2.034	1.643	1.932	1.620	2.012	1.711	1.214
Unemployment	1.324	1.366	0.279	0.260	0.301	0.265	0.254	0.239	0.492
EUR/USD	0.022	0.026	0.028	0.040	0.029	0.042	0.034	0.051	0.040
JPY/USD	0.035	0.035	0.033	0.044	0.035	0.045	0.041	0.051	0.046
UKP/USD	0.020	0.021	0.017	0.026	0.018	0.027	0.021	0.034	0.029

Note: For details see footnotes of Tables 2 and 3. However, here $T_{\text{init}} = 48$.

Table 11: Test for predictive accuracy: CGSL against DGLS and constant parameter modes for alternative forecast sample

	CGSL against DGLS							
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	AR(4)X	X
Inflation	-3.945	-7.607	-7.168	-5.951	-8.194	-4.260	-3.352	-0.724
3 month yields	1.103	0.719	-1.307	-1.557	1.851	-1.402	-1.079	1.520
10 year yields	0.168	-1.425	-1.684	-1.747	-1.917	-1.156	-0.840	-1.840
Corp bond yields	-1.487	1.948	-1.351	0.258	-1.100	-2.267	-2.385	-1.929
GDP growth	2.554	3.818	-3.023	-3.672	-1.960	1.296	-2.309	2.275
Unemployment	-4.243	-0.308	-1.835	-3.702	-3.110	-3.097	-2.581	-0.270
EUR/USD	0.716	-4.651	-4.728	-3.615	-3.150	-4.435	-3.760	2.542
JPY/USD	0.986	-0.305	-3.358	-2.487	-3.897	-4.189	-4.748	-3.328
UKP/USD	1.176	1.839	2.077	0.334	-1.064	-0.743	-2.075	1.877
	CGSL against constant parameter models							
	AE	AR(1)	ARX(1)	AR(2)	ARX(2)	AR(4)	AR(4)X	X
Inflation	-8.638	-10.106	-9.519	-9.637	-9.093	-5.511	-4.222	-6.014
3 month yields	1.095	1.251	1.366	1.287	1.515	1.482	1.597	1.180
10 year yields	0.071	-1.510	-2.045	-2.677	-3.098	-2.233	-1.968	-1.009
Corp bond yields	-2.329	1.582	-1.637	0.024	-1.314	-2.457	-2.096	-2.509
GDP growth	2.451	3.869	-2.849	-3.676	-1.594	1.330	-1.433	-4.779
Unemployment	-5.859	2.246	-1.351	-2.218	-1.269	-1.621	-1.322	-2.329
EUR/USD	-1.333	-4.738	-5.746	-5.812	-5.444	-4.424	-4.792	1.259
JPY/USD	1.006	-5.189	-4.435	-4.610	-4.880	-5.169	-4.721	-4.143
UKP/USD	1.092	1.302	1.485	1.652	0.331	-1.024	-1.260	1.576

Note: See footnote of Table 6. However, here $T_{\text{init}} = 48$.